[CONTRIBUTION FROM THE GIBBS CHEMICAL LABORATORY OF HARVARD UNIVERSITY]

Reactions of Methylene. I. Ethylene, Propane, Cyclopropane and *n*-Butane

BY H. M. FREY[1] AND G. B. KISTIAKOWSKY

RECEIVED JULY 15, 1957

Methylene, produced photochemically from ketene in the gas phase, substitutes into the primary and the secondary C–H bonds of propane and butane with a probability ratio of 1:1.7 at room temperature. This ratio can be increased by adding argon or carbon dioxide to the mixture. With ethylene methylene forms propylene and cyclopropane, with a reaction probability ratio of 1:6. This ratio can be greatly increased by adding carbon dioxide. The energy rich, "hot" cyclopropane, formed in this reaction, isomerizes unimolecularly into propylene, after a lifetime which is about 1/80 as long as that observed in its thermal isomerization. With clyclopropane methylene forms exclusively methylcyclopropane. The latter isomerizes into butenes after a lifetime comparable to that of cyclopropane. In the reactions of methylene with the alkanes ethane is a by-product; in the reaction with ethylene the main by-products are ethane and *n*-butane. Small additions of oxygen suppress completely the formation of these by-products, although its effect on the main reactions is only minor. The mechanisms of these several reactions are discussed, one of the conclusions being that methylene generated photochemically is a "hot" molecule, having vibrational energy in excess of the thermal equilibrium value. It reacts prior to losing its excess energy, *i.e.*, reacts very fast, except possibly in presence of inert gases to which it can lose its excess energy on collisions.

The chemical properties of methylene, usually generated by decomposition of ketene or diazomethane, have been studied by many investigators. The results are not altogether concordant and rather than to review the literature here, it seems preferable to postpone the discussion of relevant information until the results of this investigation of several reactions of methylene, generated photochemically from ketene, have been presented.

Experimental Details

Experiments were made in a usual all-glass system with good pumping to make sure that oxygen was excluded. The desired quantities of those gases which were to be used for an experiment were introduced into a vessel of known volume, pressure measured and the gases condensed one by one into a small side tube attached to the reaction vessel. The contents of the tube were then rapidly evaporated into a cylindrical reaction vessel of 1.8 cm. diameter and 40 cc. volume. A sealed-in-glass magnetic stirrer in the vessel ensured good mixing. The Pyrex reaction vessel was thermostated at room temperature by a water jacket.

The light source was a fairly high pressure quartz mercury arc[2] which gave the strongest emission at 3130 Å., a slightly weaker 3650 line group and still weaker 3022 Å. The resonance line was weak and strongly reversed. Making allowance for the absorption spectrum of ketene[3] and the quantum yields of its decomposition at higher pressures[4] it is concluded that decomposition was principally due to the 3130 Å. line group. A few runs were made with a quartz reaction vessel, using both the unfiltered radiation of the mercury arc and the focally isolated radiation, in the region of 2100 Å., of a condensed spark between zinc electrodes.[5] In the light of the experience gained during the experiments it now seems that it would have been an advantage to use filtered radiation in the region of 2700 Å., where the quantum yield of primary decomposition of ketene is more certainly known to be pressure independent.[4]

Ketene was prepared by a method similar to that of Jenkins[6] from acetic anhydride and was purified by fractional distillation in a constant temperature-variable pressure still with wire spiral packing. It was stored as a gas. Prior to use it was condensed by liquid nitrogen and allowed to evaporate from Dry Ice. Other gases used were taken from small commercial cylinders of highest available purity grade. They were not purified, except for condensation in a trap and the pumping off of permanent gases, where practicable.

In the experiments on ketene–ethylene mixtures, the contents of the reaction vessel, after photolysis, were passed through a trap cooled in liquid nitrogen and non-condensables discarded. The condensable fraction was then passed through a small tube of Ascarite, to remove ketene. The residue was subjected to vapor chromatography and/or mass spectrometric analysis. When CO_2 was added to these mixtures, the procedure was the same.

In experiments with propane, butane and cyclopropane the freezing out with a liquid nitrogen trap was omitted and the reaction mixture analyzed directly with the vapor chromatograph. When argon was added to these mixtures, the liquid nitrogen trap was used again.

Two vapor chromatographs were used. One was constructed especially for these experiments, the other, of higher sensitivity, was loaned to us by Mr. K. Sauer. Columns 6 ft. long, thermal conductivity detectors and a Brown recorder were employed. Separated components of a mixture could be frozen out and isolated for a mass spectrometric analysis. The columns were calibrated with known mixtures and heights of peaks and half-widths were used for quantitative measurements because the shape of the peaks was very reproducible.

The Results

Reaction with Ethylene.—In view of the finding that the attack of methylene on 1,2-dimethylethylene yields dimethylcyclopropane,[7] the reaction of methylene with ethylene was reinvestigated[8] at higher pressures in order to ascertain whether the cyclopropane that might be formed in the first stage could be stabilized by collisions. A ketene–ethylene mixture in the ratio 1:7.5 was used and the reaction allowed to proceed to about 25% decomposition of ketene. The same exposure to the radiation of the mercury lamp was required to accomplish this, regardless of pressure, which indicates a quantum yield independent of total pressure. In the mixtures used, from 95 to 89% of methylene should react with ethylene, according to whichever information in the literature one chooses.[8,9]

An alumina column, maintained at 75°, cleanly separated the C_3 fraction from the other components of the mixture. This fraction was analyzed mass spectrometrically, using mass peaks 42 and 41, whose relative intensities are quite different for propylene and cyclopropane.[10] The correctness

---

[1] Commonwealth Fund fellow. Department of Chemistry, University of Southampton, England.

[2] Actinic-100, from O. Kerber, 7920 Fourth Avenue, New York City.

[3] G. B. Kistiakowsky and B. H. Mahan, *J. Chem. Phys.*, **24**, 922 (1956).

[4] A. N. Strachan and W. A. Noyes, Jr., THIS JOURNAL, **76**, 3258 (1954).

[5] We wish to thank Dr. R. Williams for making available to us this light source.

[6] A. D. Jenkins, *J. Chem. Soc.*, 2563 (1952).

[7] P. S. Skell and R. C. Woodworth, THIS JOURNAL, **78**, 4496 (1956).

[8] G. B. Kistiakowsky and N. W. Rosenberg, *ibid.*, **72**, 321 (1950).

[9] R. A. Holroyd and W. A. Noyes, Jr., *ibid.*, **78**, 4831 (1956).

[10] NBS Tables of relative ion intensities.

of the analysis was checked with known mixtures. It was found that cyclopropane was in fact formed, its yield being pressure dependent. The asymptotic value of the ratio (cyclopropane)/(cyclopropane + propylene) was found to tend to 0.62 rather than unity at high pressures and it was concluded[11] that this is due to a simultaneous attack of methylene on the carbon–hydrogen bonds of ethylene, leading directly to propylene.

Experiments were then undertaken to investigate the effect of inert gases on the reaction. Carbon dioxide was chosen because of ease of handling in the apparatus and easy removal by Ascarite. Unfortunately, after these experiments were completed, it was established[12] that carbon dioxide is not truly inert, although the rate of its reaction with methylene is only one-fiftieth to one-eightieth of that of ketene. This small reactivity (especially when very large excesses of CO_2 were used) leads to a possible ambiguity in the interpretation of the results. The gas chromatographic procedure used did not contemplate a quantitative measurement of the absolute amounts of propylene and cyclopropane formed but only of their ratio. However, from the relative magnitudes of C_3 peaks obtained in various experiments it can be stated that even in experiments with the highest mole fractions of carbon dioxide, the total amounts of C_3 hydrocarbons formed were no less than 70% of those formed in absence of CO_2.

Figure 1 shows the ratios (cyclopropane)/(cyclopropane + propylene) plotted against total pressure in ketene–ethylene and in ketene + ethylene + carbon dioxide mixtures of compositions indicated in the legend.

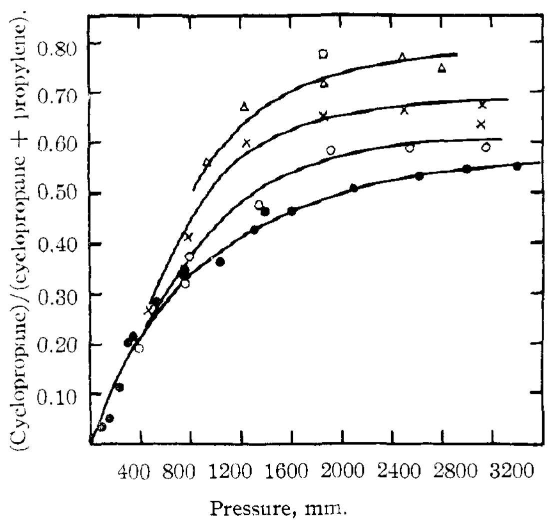

Fig. 1.—Fractional yield of cyclopropane in the reaction of methylene with ethylene, as a function of total pressure. Composition of ketene:ethylene:carbon dioxide mixtures: dots: 1:7.5:0; circles: 1:7.5:30; crosses: 1:7.5:67.5; triangles: 1:7.5:143; square: 1:7.5:292.

A few experiments were made also in three-component mixtures of ketene + ethylene + argon, the results being shown in Table I. The same table includes also the results of a few experiments made with small additions of oxygen.[5]

TABLE I

THE YIELD OF CYCLOPROPANE IN THE REACTION OF METHYLENE WITH ETHYLENE

Composition (parts)				Total pressure, mm.	$\frac{\text{Cyclopropane}}{\text{Cyclopropane + propylene}}$
CH_2-CO	C_2H_4	A	O_2		
1	7.5	30	..	2200	0.473[a] 0.467[a]
1	7.5	67.5	..	2200	.473
1	7.5	105	..	2200	.477
1	7.5	30	..	440	.178
1	7.5	30	0.6	440	.111
1	7.5	30	.5	440	.117
1	7.5	...	.5	1750	.445
1	7.5	...	.75	1750	.449

[a] Separate analyses on portions of the same reaction mixture.

In the reaction of methylene with ethylene there are formed as minor products considerable quantities of ethane and butane. The C_4 fraction, analyzed vapor chromatographically, was found to contain less than 10% of isobutane, the main component being *n*-butane. Table II shows that the ratio (*n*-butane)/(cyclopropane + propylene) does not depend significantly on either the total pressure or on the mole fraction of argon present, within the rather large experimental error in determining the amount of butane formed. In analyzing the reaction mixtures from experiments with oxygen, shown in Table I, search also was made for *n*-butane and ethane but none was found. This means that the presence of oxygen reduced their yields by more than a factor of ten. Oxygen had an undetectable effect (less than 30%) on the total yield of cyclopropane plus propylene.

TABLE II

THE YIELD OF *n*-BUTANE IN THE REACTION OF METHYLENE WITH ETHYLENE

Composition (parts)			Total pressure, mm.	$\frac{n\text{-Butane}}{\text{Cyclopropane + propylene}}$
CH_2-CO	C_2H_4	A		
1	7.5	30	430	0.131
1	7.5	67.5	782	.159
1	7.5	67.5	2100	.111
1	7.5	105	2100	.081
1	7.5	67.5	2200	.129

Reaction with Cyclopropane.—Methylene produced by the photolysis of diazomethane is known to attack carbon–hydrogen bonds[13] and the experiments with ethylene show that methylene generated from ketene acts similarly. The attack of methylene on cyclopropane was accordingly investigated, in the expectation that "hot" methylcyclopropane would be formed, which would then isomerize into butenes unless stabilized by collisions.

Ketene and cyclopropane mixtures in the ratio 1:10 were used and the photochemical reaction was again carried out to about 25% decomposition. The analysis of the products was carried out gas chromatographically, with the alumina column

---

[11] H. M. Frey, THIS JOURNAL, **79**, 1259 (1957).

[12] G. B. Kistiakowsky and K. H. Sauer, to be published as part 2 of this series.

[13] W. E. Doering, R. G. Buttery, R. G. Laughlin and N. Chaudhuri, THIS JOURNAL, **78**, 3224 (1956).

operated at both 90 and 125°. This column cleanly resolved the peak due to methylcyclopropane. This compound was identified by (a) its mass spectrum, which was that of a C_4H_8 compound, but with an intensity distribution of several fragment ion masses which did not correspond[10] to any of the butenes or to cyclobutane; (b) its infrared spectrum which showed three distinct C–H stretching vibrational frequencies; and (c) its elution time, which correlated properly with that of cyclopropane.[12] The alumina column used did not resolve the butenes into separate peaks, a rather broad asymmetric peak being observed which meant that more than one isomer was formed. This made the analysis rather uncertain and no claims of accuracy for the ratios (methylcyclopropane + butenes)/(methylcyclopropane) can be made. These ratios are plotted against inverse pressure in Fig. 2 and are very suggestive that at infinite pressure the yield of methylcyclopropane would be 100%.

Reaction with Propane.—In their work on the attack of the carbon–hydrogen bond by methylene in the liquid phase, Doering and co-workers[13] have shown that this reaction is quite indiscriminate; irrespective of whether the hydrogen is attached to the primary, secondary or tertiary carbon, the attack occurs at the same rate. To decide whether this was true for methylene produced from ketene in the gas phase, experiments were made with propane. The attack on the primary hydrogen in propane yields n-butane, whereas the attack on secondary hydrogen results in the formation of isobutane. Once again the analysis was carried out using the alumina chromatographic column, operated at 125°. The peaks due to the two butanes were fully resolved and were identified mass spectrometrically.

The ratio of isobutane to n-butane was found to be independent of total pressure over a fairly wide range (except possibly at the lowest tried) and also independent of the extent of the decomposition of ketene, varied from 10 to 85%. The results are shown in Table III. Where two values are bracketed against the same pressure, the two sets of data were obtained by a division of the gas sample after the photolysis and separate analysis. The differences between such results give an idea of the experimental errors involved. It will be noted that some of the experiments were made in Pyrex vessels, others in quartz, exposed to the total radiation of the mercury arc. Two experiments were made with radiation of 2100 Å. from the zinc spark.

The alumina column used permitted also the measurement of ethane, which is a by-product in this reaction. The last column of Table III shows the ratio of ethane to total butanes produced.

The effect of added gases on the products of reaction between methylene and propane also was investigated and the results are shown in Table IV.

In the system just discussed there is a competition between the ketene and the propane for methylene. The extent of the reaction with ketene can be calculated from the quantity of ethylene formed by the over-all reaction

$$CH_2 + CH_2CO = C_2H_4 + CO$$

and therefore some chromatographic analyses included the determination of ethylene. In Fig. 3 the ratio of ethylene to total butanes is plotted as a function of percentage decomposition of ketene. As is to be expected, this ratio decreases with % decomposition, due to depletion of ketene. Extrapolation to zero decomposition shows that methylene reacts with ketene approximately 1.1 times as fast as it does with propane.

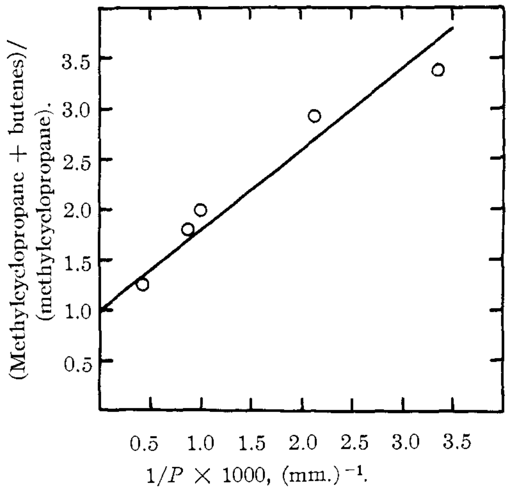

Fig. 2.—A plot of the inverse of the fractional yield of methylcyclopropane against the inverse of total pressure in 1:10 ketene:cyclopropane mixtures.

Methane was also noted as a product of the reaction with propane but could not be determined with the alumina column. Two runs were analyzed with the aid of an activated carbon column operated at 125°. A good methane peak was obtained, which indicated that the yield of methane was ap-

TABLE III

THE REACTION OF METHYLENE WITH PROPANE IN THE 1:8 KETENE:PROPANE MIXTURES

Total pressure, mm.	Dec. of ketene, %	$\frac{nC_4H_{10}}{iso\text{-}C_4H_{10}}$	$\frac{C_2H_6}{Total\text{-}C_4H_{10}}$
45	25	1.90	0.072
48	25[b]	1.80	
56	25[b]	1.79	
90	25[a]	1.72	.066
		1.78	
395	25	1.74	.068
		1.77	
600	17.0[a]	1.77	.063
600	29.0[a]	1.75	.061
		1.73	
600	56.5[a]	1.71	.067
		1.73	
600	85.0[a]	1.72	.065
		1.64	
666	25	1.62	
		1.66	
962	25	1.68	.065
		1.75	
1560	25[a]	1.72	.062

[a] Quartz reaction vessel and the mercury arc. [b] Quartz reaction vessel and the zinc spark.

TABLE IV

THE EFFECT OF OTHER GASES ON THE REACTION OF METHYLENE WITH PROPANE

Composition (parts)					Total pressure, mm.	Yields	
CH_2-CO	C_3H_8	A	CO_2	He		$\frac{n\text{-}C_4H_{10}}{\text{iso-}C_4H_{10}}$	$\frac{C_2H_6}{\text{total-}C_4H_{10}}$
1	8	64	..	..	1595	1.43	
1	8	64	..	..	1590	1.48	0.127
						1.51	
1	8	96	..	..	1974	1.40	.111
						1.48	
1	8	..	64	..	1593	1.32	.122
						1.37	
1	8	..	..	64	700	1.61	
						1.59	

proximately 15% of that of ethane and therefore approximately 1% of the butanes.

Reaction with *n*-Butane.—In the photolysis of mixtures of ketene and *n*-butane, present in the ratio 1:8, the products of the reaction were found to be normal and isopentane, identified by chromatography with the alumina column at 125°. Small amounts of ethane were also observed to be formed. In addition there were found ethylene and propylene, both undoubtedly resulting from the reactions of methylene with ketene and with ethylene formed thereby. Table V shows the relative yields of the pentanes and of ethane.

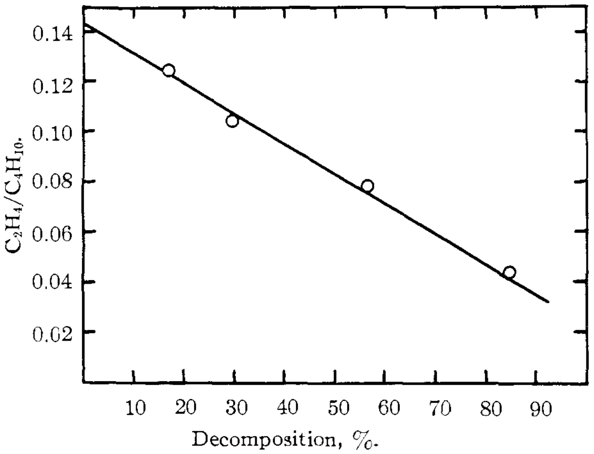

Fig. 3.—A plot of the relative yield of ethylene as a function of the degree of decomposition of ketene in 1:8 ketene: propane mixtures.

TABLE V

THE REACTION OF METHYLENE WITH *n*-BUTANE IN THE 1:8 KETENE:BUTANE MIXTURES

Total pressure, mm.	$\frac{n\text{-}C_5H_{12}}{\text{iso-}C_5H_{12}}$	$\frac{C_2H_6}{\text{Total-}C_6H_{12}}$
570	1.13	0.065
	1.05	
800	1.13	.067

Discussion of the Results

The data presented in the preceding section deal with several reactions of methylene and reveal considerable complexity in all of them. Somewhat simpler is the substitution of methylene into the carbon–hydrogen bond[14] and this subject will be considered first.

Reactions of Methylene with Propane and Butane.—Table III shows that the relative yield of *n*- and isobutanes is unaffected by total pressure and by the degree of decomposition of ketene. The results are therefore entirely consistent with the conclusion of Doering that methylene adds directly into the C–H bond. This reaction is accompanied by a release of over 90 kcal. energy, if the heat of formation of methylene is 84 kcal. (see below). Such energy is sufficient to break a carbon–carbon bond and to cause a number of other reactions in the hydrocarbon molecules, but the independence of the relative yields from total pressure indicates that such reactions are not taking place, presumably because the energy is lost on collisions before anything else happens.[15] In contradistinction to the conclusion of Doering that the attack of methylene on primary and secondary carbon–hydrogen bonds is entirely indiscriminate, present experiments show significant discrimination. The mean value of the ratio of *n*-butane to isobutane obtained from all the results in Table III is 1.73, whereas if the attack were statistically random, the ratio would be 3.0. The ratio actually observed implies that under the conditions of experiments of Table III methylene attacks the secondary hydrogen bonds of propane 1.7 times as fast as it attacks the primary bonds. This is a characteristic property of methylene produced from ketene under the conditions of these experiments, because the ratio of *n*-pentane to isopentane in experiments with *n*-butane is 1.1, whereas indiscriminate attack calls for a ratio of 1.5. Here methylene attacks secondary hydrogen bonds 1.67 times as fast as it does the primary ones. Within experimental errors this is the same value as obtained in the more extensive runs with propane.

The data of Doering and co-workers[13] indicate that the ratio of reaction probabilities with a C–H bond and with a C=C bond is about 1:6,[16] whereas it is now found that methylene generated from ketene attacks propane and ketene almost equally fast, so that the above ratio is 1:8, if the primary C–H, bonds are considered.

In a recent note Knox and Trotman-Dickenson[17] report the results of their experiments on the attack of methylene generated from ketene on several paraffin hydrocarbons. Ratios of reaction probabilities which they show are very close to those here presented, proving further that these results are not due to experimental errors.

Because of the independence of these ratios from total pressure, their deviation from the results of Doering is not likely to be due to a change from the liquid to the gaseous state. A difference in the properties of methylene is the alternative explanation. Experiments with added gases were undertaken to test this point. Addition of carbon dioxide is seen indeed to alter the observed ratio of the

---

(14) M. Vanpee and F. Grard, *Bull. soc. chim. Belg.*, **60**, 208 (1951).

(15) Except possibly at the lowest pressures.

(16) We are indebted to Prof. Doering for pointing out that this is the correct value derived from their data, rather than 1:3, as stated in their paper.

(17) J. H. Knox and A. F. Trotman-Dickenson, *Chemistry & Industry*, 731 (1957).

butanes to a large degree. Argon has the same effect; although smaller, it is definitely outside the limits of experimental error. Effect of helium is still smaller and cannot be regarded as definitely proven. All these gases act in the direction of increasing the discrimination, that is, they favor the attack on the secondary C–H bonds. Two possibilities occur: (a) the inert gases stabilize more effectively the energy-rich branched molecules than they do the normal hydrocarbons; (b) methylene, when formed from ketene, receives energy in excess of thermal energy; the inert gas molecules remove all or some of it before the reaction with the hydrocarbons; the loss of energy by methylene results in a more discriminating attack on different carbon–hydrogen bonds. The first possibility can be ruled out because in absence of inert gases the total pressure has no effect on the relative yields. The correct explanation is therefore that methylene, when formed in the photochemical decomposition of ketene, possesses excess energy, which alters its relative reaction probability toward different C–H bonds and the C=C bond.

There is some indication that the configuration of the free CH_2 and of CH_2 in ketene is not identical,[18] which makes the vibrational excitation of methylene in the process of decomposition of ketene more probable. Moreover, it has been recently demonstrated that ethylidene, resulting from the photochemical decomposition of methyl ketene, carries away excess energy[2] and an extension of this finding to methylene is very plausible.

The effect of carbon dioxide on relative reaction probabilities may be more complex than that of argon, because the former is not wholly inert, but reacts, albeit slowly, with methylene.[12] Two possibilities exist: (a) carbon dioxide, while occasionally reacting with methylene, also acts as an effective quencher, removing excess energy from other methylene molecules and so altering their reactivity; (b) the methylene molecules, when formed from ketene, have a wide spectrum of energies and carbon dioxide reacts only with the more energetic ones. The remaining methylenes show greater discrimination in their attack on C–H bonds. The latter hypothesis is made less probable by a study of the effect of wave length of radiation on the methylene–carbon dioxide reaction[12] but cannot be wholly ruled out. It might be added that we have been unable to construct a mechanism which accounts for the observations, but which replaces the notion of "hot" methylene by a combination of reactions of thermally equilibrated methylene and of electronically excited ketene molecules.

Spectroscopic information is inconclusive as regards the existence of "hot" methylene. The observations of Doering and co-workers on indiscriminate attack by methylene generated from diazomethane, which suggest that this methylene has even more energy, are understandable because the diffuse absorption spectrum of diazomethane[19] extends to 4700 Å. as against 3800 Å. of ketene. In both sets of experiments the unfiltered radiation from mercury arcs passing through Pyrex was used. The photons absorbed by diazomethane have therefore as much as 30 kcal. excess energy over the dissociation energy. In the case of ketene this excess is of the order of 15 kcal. But what fractions of these excess energies are taken up by methylene, depends on the details of the dissociation processes, which are not known. The ratio of reaction probabilities with secondary and primary C–H bonds observed with a quartz reaction vessel (Table III) was not significantly different from that observed with Pyrex. This is not surprising because, when allowance is made for the intensity distribution of the arc used and the absorption spectrum of ketene, it is found that in both instances the 3100 Å. wave length region was the main contributor to the formation of methylene. The experiments (Table III) with the 2100 Å. line group which show the same discrimination between primary and secondary C–H bonds present a real difficulty from the point of view of the "hot" methylene hypothesis, because the photons involved have some 55 kcal. excess energy. The absorption of this radiation, however, brings ketene molecules into a different electronic state[20] than the absorption in the 3100 Å. band. Further investigation is needed to determine what is the state of methylene resulting from the dissociation of ketene molecules in this electronic state.[12,21]

Reactions with Ethylene and with Cyclopropane.—As already stated,[11] the experiments with ketene + ethylene mixtures prove that methylene reacts with ethylene in two ways. One, the attack on the C–H bond, results in direct formation of propylene, the other, an addition to the double bond, produces cyclopropane with so much excess energy over that required for isomerization into propylene that this reaction occurs after a lifetime which is only one-eightieth of that of thermally activated cyclopropane molecules, unless the excess energy is removed by collisions.

A closer inspection of the curve obtained with ketene + ethylene mixtures shows that the pressure dependence of the yield of cyclopropane is of a somewhat higher order than the simple theory of unimolecular decompositions would lead one to expect. A possible explanation is that the deactivation of the very "hot" cyclopropane molecules occurs stepwise (*e.g.*, by the successive losses of single vibrational energy quanta), each step resulting in a molecule of longer lifetime, until isomerization becomes totally impossible. The experimental results can be fitted by assuming two such steps

$$\begin{aligned}
&\mathrm{CH_2 + C_2H_4 \longrightarrow CH_3CH{=}CH_2};\ k_1\\
&\mathrm{CH_2 + C_2H_4 \longrightarrow c\text{-}C_3H_6^{**}};\ k_2\\
&\mathrm{c\text{-}C_3H_6^{**} + M \longrightarrow c\text{-}C_3H_6^{*} + M};\ k_3\\
&\mathrm{c\text{-}C_3H_6^{*} + M \longrightarrow c\text{-}C_3H_6 + M};\ k_3\\
&\mathrm{c\text{-}C_3H_6^{**} \longrightarrow CH_3CH{=}CH_2};\ k_4\\
&\mathrm{c\text{-}C_3H_6^{*} \longrightarrow CH_3CH{=}CH_2};\ k_5
\end{aligned}$$

Defining by r_p the (cyclopropane)/(cyclopropane + propylene) ratio at a pressure p and by r_∞ the

---

[18] (a) W. F. Arendale and W. H. Fletcher, *J. Chem. Phys.*, **26**, 793 (1957); (b) J. Duchesne and L. Burnelle, *ibid.*, **21**, 2005 (1953); (c) P. Vankateswarlu, *Phys. Rev.*, **77**, 676 (1950).

[19] F. W. Kirkbride and R. G. W. Norrish, *J. Chem. Soc.*, **119** (1933).

[20] P. Wagner and A. B. F. Duncan, *J. Chem. Phys.*, **21**, 516 (1953).

[21] H. Gesser and E. W. R. Steacie, *Can. J. Chem.*, **34**, 113 (1956).

same ratio at infinite pressure, one obtains from the above equations the expression

$$(\mathrm{M})^2\left[\frac{1}{r_\mathrm{p}} - \frac{1}{r_\infty}\right] = \left(\frac{k_4 + k_5}{k_3}\right)\frac{1}{r_\infty}(\mathrm{M}) + \frac{k_4k_5}{k_3}\frac{1}{r_\infty} \quad (1)$$

The plot of the left-hand side of this equation against total pressure does give a good fit to a straight line using data of Fig. 1. Unfortunately, the slope and the intercept of the line are very sensitive to the value of r_∞, and this is not known with adequate precision to gain quantitative information on the individual rate constants k_3, k_4 and k_5.

A self-consistent calculation of the energy content of "hot" cyclopropane may be carried out on the basis of the classical theory of unimolecular decompositions of Slater[22] and experimental data on the thermal isomerization of cyclopropane.[23] Substituting $n = 13$, $E_0 = 65$ kcal. and $E_1 - E_0 = 19.5$ kcal. (the classical energy of 13 oscillators at 765°K.) into the equation

$$80 = \left[\left(\frac{E_2 - E_0}{E_1 - E_0}\right)\frac{E_1}{E_2}\right]^{12} \quad (2)$$

one obtains as the energy of the photochemically produced cyclopropane molecules $E_2 = 97$ kcal. This value, however, is in doubt, because the 19.5 kcal. used in deducing it is a fiction, arising from the classical treatment adopted. The total vibrational energy of average cyclopropane molecules at 765°K. is about 5 kcal. The real value of E_2 may be presumed therefore to be lower than 97 kcal., but it does not appear profitable at present to attempt more elaborate calculations, on account of great uncertainties as to the heat of the reaction $C_2H_4 + CH_2 = \text{cyclo-}C_3H_6$. The range of values for the heat of this reaction appears to be from 77 to 92 kcal., 84 being most probable. This is to be compared with an equally uncertain value of 97 kcal. from the ratio of lifetimes. Some indication of excess energy in methylene thus exists, but it is by no means proven by these considerations.

Experiments on the ethylene–methylene reaction with added gases (Fig. 1 and Table I) provide some further information on the problems discussed above. The ratio of reaction probabilities with the C–H and C=C bonds is about 6 from the limiting yield (62%) of cyclopropane in absence of added gases, essentially the same value as obtained earlier on comparing the reactions of methylene with propane and with ketene. The addition of carbon dioxide increases this ratio to about 16 when methylene suffers on the average 30 collisions with carbon dioxide before colliding with ethylene. The trend of the limiting yields of cyclopropane with the mole fraction of carbon dioxide is such that with an infinite excess of carbon dioxide the only observable reaction would be that with the double bond of ethylene. These findings certainly support the hypothesis of "hot" methylene, although the reactivity of carbon dioxide makes the exact mechanism of its action slightly uncertain (see above). It is noteworthy that while carbon dioxide is rather ineffective in changing the limiting yield of cyclopropane and argon has no observable effect in this direction, the former is as effective as ethylene (Fig. 1) and argon is about 50% as effective (Table I) in stabilizing the energy rich cyclopropane molecules. Thus it would appear that the latter energy transfer process occurs on most collisions, whereas the former does not. Table III showed that helium also is quite ineffective in altering the reactivity of methylene. This eliminates translational energy of freshly formed methylene as the principal cause of its special reactivity. On the other hand it is well known from sound dispersion and shock wave relaxation studies that low frequency vibrational energy of complex molecules is far more readily transferred on collisions than is that of simple molecules with high vibrational frequencies. The different behavior of "hot" cyclopropane and "hot" methylene is in line with these general findings.

The fractional cyclopropane yield *versus* pressure curves obtained with added carbon dioxide show even sharper bends (Fig. 1) than the curve in the absence of added gases. They can be still represented by eq. 1 if appropriate r_∞ are chosen but the scatter of experimental data is just too large and their number too small to permit a real test of eq. 1 and so this entire question must be left in abeyance.

The results obtained on the reaction of methylene with cyclopropane, although not very exact, justify some consideration. In agreement with the finding of Doering[13] that methylene does not attack the single carbon–carbon bond, the limiting yield of methylcyclopropane is seen to be near 100%. The lifetime of the "hot" methylcyclopropane is within a factor of two equal to that of cyclopropane, which is quite interesting because the former has a larger number of internal degrees of freedom but also contains some 6 kcal. greater excess energy than does cyclopropane. Apparently these two factors, which affect lifetimes in unimolecular reactions in opposite directions, accidentally cancel out.

Formation of By-products and the Effect of Oxygen.—In the reaction of methylene with ethylene and with alkanes, there are formed considerable quantities of ethane and much smaller quantities of methane. In the reaction with ethylene *n*-butane also is formed. It is noteworthy that the relative yield of ethane is the same whether propane or butane is used. To explain these findings it is necessary to postulate an additional reaction of methylene, the abstraction of a hydrogen atom with the resultant formation of a methyl radical.[21] A second abstraction reaction[24] results then in the formation of methane; the association of two methyl radicals produces ethane, the most abundant by-product. Finally, the formation of *n*-butane in the ketene–ethylene mixtures is probably the result of the sequence

$$CH_3 + C_2H_4 = CH_3CH_2CH_2$$
$$CH_3 + C_3H_7 = C_4H_{10}$$

The complete elimination of ethane and butane in ketene–ethylene mixtures when oxygen was also

---

[22] N. B. Slater, *Phil. Trans.*, **246A**, 57 (1953); *Proc. Roy. Soc.* (*London*), **218A**, 224 (1953).

[23] H. O. Pritchard, R. G. Sowden and A. F. Trotman-Dickenson, *ibid.*, **217A**, 563 (1953).

[24] E. W. R. Steacie, "Atomic and Free Radical Reactions," Reinhold Publ. Corp., New York, N. Y., 1954.

present, is strongly in favor of the proposed mechanism involving methyl radicals, because the latter are known[24] to react very rapidly with oxygen. On the other hand, the increase in the yield of ethane in the presence of inert gases (Table IV) eludes an explanation. The experiments with oxygen were the last to be performed before the unavoidable termination of this research and the effect of oxygen in changing the ratio between cyclopropane and propylene at low but not at higher pressures (Table I) could not be adequately explored.

There are thus several loose ends to this investigation which need further exploration before the reaction mechanisms here discussed can be considered as conclusively proven. What is definitely established by these experiments is the complexity of even supposedly simple reactions of methylene. The contradiction between the present experiments and those of Doering, as well as the effect of inert gases in modifying the reactivity of methylene, has only one logically tenable explanation. It is that methylene formed photochemically has an excess of energy over the equilibrium thermal value and that it reacts, in absence of inert gases, before this energy is dissipated on collisions. This means that "hot" methylene reacts very fast. What the reactivity of methylene thermally equilibrated at room temperature is, still remains to be determined.

CAMBRIDGE, MASS.

[CONTRIBUTION FROM THE DEPARTMENT OF CHEMISTRY, UNIVERSITY OF WISCONSIN]

Studies of the Enzyme Fumarase. VII.[1] Series Solutions of Integrated Rate Equations for Irreversible and Reversible Michaelis–Menten Mechanisms[2]

BY ROBERT A. ALBERTY AND BARBARA M. KOERBER

RECEIVED MAY 18, 1957

The integrated rate equations for the Michaelis–Menten mechanism and for simple reversible mechanisms have been expanded as power series in time to determine the relations between the intercepts and initial slopes of plots of $(\mathrm{P})/t$ or $-(1/t)\ln[1-(\mathrm{P})/(\mathrm{P})_{\mathrm{eq}}]$ *vs.* t and the kinetic parameters. The magnitudes of the experimental initial slopes for the fumarase reaction are in agreement with theory. It is shown that the initial slopes of these plots, as well as the intercepts, are useful in determining the kinetic parameters for enzymatic reactions. By calculating the coefficient of the t^2 terms in the series it is shown that the onset of appreciable deviations from linearity occurs at the same time for the two types of plots.

Introduction

The kinetic parameters for enzymatic reactions are generally calculated from steady-state velocities extrapolated to zero time to eliminate the effect of accumulation of product on the rate. Since the steady-state velocity may decrease rather rapidly as the reaction proceeds, there may be some difficulty in making the extrapolation to zero time. We have therefore investigated the relation between the slopes of plots of $(\mathrm{P})/t$ or $-(1/t)\ln[1-(\mathrm{P})/(\mathrm{P})_{\mathrm{eq}}]$ *vs.* t during the steady state and the steady state kinetic parameters, where (P) is the concentration of product and $(\mathrm{P})_{\mathrm{eq}}$ is the equilibrium concentration. Such plots may be represented by power series expansions

$$(\mathrm{P})/t = a\{1 + bt[1 + ct + \ldots]\} \qquad (1)$$

$$-(1/t)\ln[1-(\mathrm{P})/(\mathrm{P})_{\mathrm{eq}}] = \alpha\{1 + \beta t[1 + \gamma t + \ldots]\} \qquad (2)$$

For a particular steady-state rate equation, $a, b, c \ldots$ and $\alpha, \beta, \gamma \ldots$ may be obtained in terms of the rate parameters by means of series expansions of the integrated rate equation. These equations make it possible to calculate the length of the period during which a linear extrapolation to $t = 0$ may be made. They also permit the calculation of kinetic parameters from the initial slopes of such plots as well as from the intercepts or make possible a check on the consistency of the rate parameters determined from initial velocities with the decrease in velocity with time.

Michaelis–Menten Mechanism

The integrated rate equation for the mechanism

$$\mathrm{E} + \mathrm{S} \rightleftarrows \mathrm{ES} \longrightarrow \mathrm{E} + \mathrm{P} \qquad (3)$$

was obtained by Walker and Schmidt[3] on the assumptions that the time required to reach the steady state is negligible, that $d(\mathrm{ES})/dt = 0$ is a sufficiently good approximation,[4] that the reaction goes to completion and the product is not inhibitory and that $(\mathrm{S})_0 >> (\mathrm{E})_0$. The integrated rate equation may be written

$$V_{\mathrm{S}}t = (\mathrm{P}) - K_{\mathrm{S}}\ln[1-(\mathrm{P})/(\mathrm{S})_0] \qquad (4)$$

where V_{S} is the maximum steady-state velocity, and K_{S} is the Michaelis constant of the substrate and $(\mathrm{S})_0$ is the initial substrate concentration. Walker and Schmidt obtained V_{S} and K_{S} for the histidase reaction from a plot of $(1/t)\ln[1-(\mathrm{P})/(\mathrm{S})_0]$ *vs.* $(\mathrm{P})/t$.

Equation 4 may be rearranged into the form of equation 1 by expanding the logarithmic term as a power series.

$$-\ln\left[1-\frac{(\mathrm{P})}{(\mathrm{S})_0}\right] = \frac{(\mathrm{P})}{(\mathrm{S})_0} + \frac{(\mathrm{P})^2}{2(\mathrm{S})_0^2} + \frac{(\mathrm{P})^3}{3(\mathrm{S})_0^3} + \ldots \qquad (5)$$

Substituting into equation 4 and rearranging we obtain

$$\frac{(\mathrm{P})}{(\mathrm{S})_0} = \frac{V_{\mathrm{S}}t}{(\mathrm{S})_0 + K_{\mathrm{S}}} - \frac{K_{\mathrm{S}}(\mathrm{P})^2}{2[(\mathrm{S})_0 + K_{\mathrm{S}}](\mathrm{S})_0^2} - \frac{K_{\mathrm{S}}(\mathrm{P})^3}{3[(\mathrm{S})_0 + K_{\mathrm{S}}][\mathrm{S}]_0^3} - \ldots \qquad (6)$$

For small extents of reaction, $(\mathrm{P})/(\mathrm{S})_0 << 1$ and the first term on the right-hand side of equation 6 may be substituted for $(\mathrm{P})/(\mathrm{S})_0$ in the second

---

[1] The preceding article in this series is R. A. Alberty, W. G. Miller and H. F. Fisher, THIS JOURNAL, **79**, 3973 (1957).

[2] This research was supported by grants from the National Science Foundation and from the Research Committee of the Graduate School of the University of Wisconsin from funds supplied by the Wisconsin Alumni Research Foundation.

[3] A. C. Walker and C. L. A. Schmidt, *Arch. Biochem.*, **5**, 445 (1944).

[4] M. F. Morales and D. E. Goldman, THIS JOURNAL, **77**, 6069 (1955).